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MU120
Open Mathematics

Preparatory Readings

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Acknowledgements

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Texts

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Illustrations

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Some uses for digital codes by Stephen Barnett

Introduction

When you hear the word 'codes', you may think of spies, secrecy and deception. But in fact a vital ingredient of contemporary life is the use of codes to transmit and process information as correctly and accurately as possible – the opposite of concealment. Examples abound in everyday life. Take that modern marvel, the compact disc (CD)—why does every one carry the words *digital audio*? What have digits (numbers) got to do with sound? If you have a cashcard, then to operate it you must enter your PIN (Personal Identification Number)—can you really be sure that the bank's computer will correctly identify your account?

If you look at any household product in your kitchen cupboard—perhaps a packet of flour or a tin of fruit—it will probably have on it a barcode, a pattern of vertical thick and thin black bars, together with a number (often consisting of thirteen digits) underneath. You may have seen the checkout operator in a supermarket using a laser reader on this barcode, and the product name and price magically appear on the checkout display panel. There must be a computer in the store which processes the information, but why does it make mistakes so rarely?

Look at any book, and on the back cover you will find a ten-digit number called an ISBN (International Standard Book Number)—this too must be some kind of identification number, but what does it mean, and why is the last digit sometimes 'X'? Newspapers and magazines carry a similar but differently designed number. So do new-style European Community passports.

The answers to all these questions are intriguing, and reveal that many of the wonders of contemporary microchip electronics rely on using the mathematics of error-detecting codes. Detecting errors is what the supermarket product barcode does—it identifies the product and price, and also detects any errors. The laser reader sends the number to the shop's computer which carries out a simple arithmetic calculation (you can easily do it yourself once you know how) and uses the last digit of the barcode to check if the number is valid—otherwise it signals an error to the checkout operator. The error may have been caused by dirt obscuring part of the barcode, or a faulty reading. You probably have seen the operator then entering the code by hand into the keyboard. Of course, it is quite easy to make another mistake when doing this—typing say '52' instead of '25'. Again, the code enables the computer to detect most errors of this type also.

The ISBN, newspaper and periodical code numbers and passport numbers are also of this type: there is one 'check digit' which, after a simple arithmetical test, reveals whether anything is wrong. If so, then a human operator has to take some action. This might merely take the form of repeating the original operation—like the situation when you ask someone to repeat what they have said when you didn't quite make it out the first time around. Verbal and written language contains a lot of structure, so that often you can understand what someone has said (or written) even if there are bits missing. The essential idea of check digits is to add *mathematical* structure, to recognize errors and distortions which inevitably occur during processing and transmission of information. This

article explores in more detail some of the applications and techniques of digital codes.

Elementary codes

How is error-detection carried out? Suppose a telecommunications engineer wishes to design a system which transmits information as reliably as possible along some communication channel. This might be a telephone line, satellite link or magnetic disc on a computer. For example, if you have a bank cashcard, you want to be sure when you key in your PIN that the bank's computer does not make any mistakes with your account.

In written language, there is often sufficient surplus information in the structure of the language itself to enable the reader to correctly guess the message even if there are spelling or other errors. For example, reading in a newspaper weather forecast the statement: 'Tomorrow there will be shwrs and wand', you are probably safe in guessing it means 'showers and wind'. However, if you received a circular advertising a cheap deal on 'slirts', but giving no other information, you have no way of knowing whether this means 'shirts' or 'skirts'. The fundamental idea of an error-detecting code is to add surplus information to the original message in the form of extra 'check' digits decided on in some cunning way so that at the receiving end an error in the original correct message can be detected.

Barcodes on household goods

If you buy a packet of flour in a branch of one of the major British supermarket chains, it usually will have a barcode on it, similar to the one shown in Figure 1.

5 000119 004048

Figure 1

The bars are read at the checkout counter by a laser scanner which works on the different ratios of the widths of the light and dark bars. The scanning of the barcode produces a number with 13 decimal digits (which are printed underneath) and uniquely identifies the product. Sometimes, if the scanner does not work properly, the checkout operator enters this number by hand into the keyboard. The price of the product is not part of the barcode. That information is held in the shop's computer, which informs the electronic cash register of the price of each item as its barcode is scanned. It is essential for there to be no mistakes, or you would be charged the wrong price for a purchase.

The barcode in Figure 1 is an example of a widely used system called the European Article Number (EAN) system, which consists of a decimal number with thirteen digits. The first two digits are allocated to countries, so '50' means 'belonging to the United Kingdom'. The next five digits '00119' give the manufacturer's number, which in this case means 'belongs to Tesco', and the next five digits '00404' give the unique number identifying the product, which here is Tesco's self-raising flour. The clever

part comes with the last 'check' digit '8', so-called because it enables to computer to check that it has the correct number for the item scanned. Here are some more EANs:

Tesco tinned peaches	5000119105400
Whitworths sugar	5013026085000
Maille mustard	3036810208050

You can see that the second Tesco product shares the same first seven digits with the flour; the sugar originates in Britain so the code begins with '50'; but the Maille mustard comes from France, so its code starts with '30'.

You can carry out the check yourself as follows:

Step 1: Label the barcode digits in turn with the numbers from 1 to 13 starting from the left, so that for the flour you get

Barcode digit	5	0	0	0	1	1	9	0	0	4	0	4	8
Reference number	1	2	3	4	5	6	7	8	9	10	11	12	13

Step 2: Add together the *odd* numbered barcode digits to get $5+0+1+9+0+0+8=23$

Step 3: Add together the *even* numbered barcode digits (that is, the ones not used so far) to get $0+0+1+0+4+4=9$ and *multiply* this sum by 3 to obtain $9 \times 3 = 27$.

Step 4: If no errors have occurred, the *sum* of the two numbers you have found in Steps 2 and 3 should be a multiple of 10—here $23+27=50=5 \times 10$.

When you know the country, manufacturer's number and product number, then you can use this process to work out the check number which gives the complete EAN. For example, with Tesco's tinned peaches:

Step 1:

5	0	0	0	1	1	9	1	0	5	4	0	?
1	2	3	4	5	6	7	8	9	10	11	12	13

Step 2: $5+0+1+9+0+4+\,?=19+\,?$

Step 3: $3 \times (0+0+1+1+5+0)=21$

The sum of these two numbers is $19+21+\,?=40+\,?$ So the check digit ? must be 0 (because if it were any of 1 to 9, the sum would not be an exact multiple of 10) which, as you can see from the short list of EANs above, is correct.

Now use the same scheme to check the EANs for the sugar and mustard, and raid your cupboards for other examples you can test.

If something goes wrong, perhaps the barcode is dirty and gives the scanner a faulty reading, or the operator makes a mistake with the keyboard entry, then the check sum in Step 4 cannot possibly be a multiple of 10. To see this, suppose the first digit 5 in the flour barcode erroneously changes to 7: the sum in step 2 will be increased by 2, and the overall sum in Step 4 will also be increased by 2 to 52, no longer a multiple of 10. You can perhaps see that whatever the amount any odd-numbered digit is changed by (and this amount is bound to be less than 10), then the sum in Step 2 is changed by the same amount; the overall sum in Step 4 is also changed by that amount, so cannot be a multiple of 10 any longer.

Now suppose the second digit 0 in the flour barcode is changed to 7: the sum in Step 3 is increased by $3 \times 7 = 21$, so the check sum in Step 4 increases to $50+21=71$, again no longer a multiple of 10. The crucial

fact here is that whatever amount any even-numbered digit changes by, the sum in Step 3 changes by three times this amount, and this product can never be a multiple of 10 (can you see why?)—hence again the check sum in Step 4 is no longer a multiple of 10.

The argument has shown that the EAN code *detects* any error in a single digit, since the check sum will not satisfy the condition in Step 4 for any faulty number. The checkout operator is alerted to the error, and simply repeats the entry for the item causing the error signal, so an incorrect charge is avoided.

Another common mistake which can occur when the checkout operator enters a code number manually is to inadvertently transpose (swap around) two adjacent digits. You have probably done this yourself with a keypad telephone. Suppose that in the flour barcode the sixth and seventh digits are accidently transposed, giving:

$$\begin{array}{ccccccccccccc} 5 & 0 & 0 & 0 & 1 & \multicolumn{2}{c}{\overbrace{9 \quad 1}^{\textit{transposed}}} & 0 & 0 & 4 & 0 & 4 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \end{array}$$

The new sum in Step 2 is now

$$5+0+1+1+0+0+8=15$$

and Step 3 gives

$$3\times(0+0+9+0+4+4)=51$$

The overall sum is 66 which is not a multiple of 10, so an error has been detected. However, if instead the first two digits were transposed, you should check that the number 0500119004048 when tested by the above procedure gives an overall total of 60, which *is* a multiple of 10! In this case, the transposition error is present in the number, but is not detected. You might like to try some of the digits in other EANs, and convince yourself that transposition errors are always detected *except* when the two adjacent digits differ by 5.

International Standard Book Number

ISBN 0-19-859640-5

9 780198 596400

Figure 2

Every book published carries its own unique identifying number like that in Figure 2, called an International Standard Book Number (ISBN), usually located in the bottom right-hand corner of the back cover. This is a 10-digit number which uniquely identifies a book. The first digit denotes the country—the UK, USA and some others have 0; the next two digits give the publisher—here 19 is Oxford University Press. The next six digits are the book number assigned by the publisher. All these first nine digits are decimal numbers (in other words, run from 0–9), but the last 'check' digit can also take the value 10, denoted by the Roman numeral X (so Roman numerals still have a practical use!). There are, in fact,

variations—some countries have a two-digit number, and some publisher numbers have more than two digits, in which case the book number has less than six digits. Incidentally, the hyphens in the ISBN are inserted purely for convenience in breaking up the number into blocks, and have no mathematical significance.

If you handle book orders, then the first thing to check is that the number you have been given is correct—that is, it is indeed a valid ISBN for a book. This is done as follows:

Step 1: Write down the ISBN as a row of digits:

0 1 9 8 5 9 6 4 0 5

Step 2: Construct two more rows of numbers beneath this row, working from left to right, according to the following rules:

(a) The first number in each new row is the same as the first digit in the ISBN:

$$\begin{array}{cccccccccc} 0 & 1 & 9 & 8 & 5 & 9 & 6 & 4 & 0 & 5 \\ 0 \\ 0 \end{array}$$

(b) The two numbers in the next column are computed by following the arrows and adding, as shown:

$$\begin{array}{ccl} 0 & 1 & \\ & \downarrow & \\ 0 \rightarrow & \boxed{1} & 0+1=1 \\ & \downarrow & \\ 0 \rightarrow & \boxed{1} & 0+1=1 \end{array}$$

Then continue in an exactly similar way to obtain the third column of numbers:

$$\begin{array}{cccl} 0 & 1 & 9 & \\ & & \downarrow & \\ 0 & 1 \rightarrow & \boxed{10} & 1+9=10 \\ & & \downarrow & \\ 0 & 1 \rightarrow & \boxed{11} & 1+10=11 \end{array}$$

and so on:

$$\begin{array}{ccccl} 0 & 1 & 9 & 8 & \\ & & & \downarrow & \\ 0 & 1 & 10 \rightarrow & \boxed{18} & 10+8=18 \\ & & & \downarrow & \\ 0 & 1 & 11 \rightarrow & \boxed{29} & 11+18=29 \end{array}$$

You should continue this process to end up with:

0	1	9	8	5	9	6	4	0	5
0	1	10	18	23	32	38	42	42	47
0	1	11	29	52	84	122	164	206	253

The final number (circled) must be a multiple of 11 for the ISBN to be correct—here $253 = 23 \times 11$. If this 'check sum' is not a multiple of 11, then at least one error has been detected.

Apply this checking method to the ISBN of any book you happen to be using at the moment. (Open University course units carry ISBNs.)

You might be wondering why the check sum has to be divisible by 11 rather than 10, as with the EAN code. The answer lies in the problem discovered when the EAN check was applied to transposition errors—not all of them were detected. The use of the number 11 for the ISBN guarantees that errors in any single digit are always detected. Moreover, the use of 11 carries a further double bonus: first, *any* transposition of adjacent digits is always detected; second, if you know that a particular single digit is in error—for example, a digit might be smudged, or obscured by a coffee stain—then this digit can actually be corrected. To see how this works, suppose that the digit 4 in the example is unknown. You can get as far as:

0	1	9	8	5	9	6	?	0	5
0	1	10	18	23	32	38			
0	1	11	29	52	84	122			

The remaining part of the check procedure is therefore:

?	0	5
38+?	38+?	43+?
160+?	198+2(?)	241+3(?)

The unknown number must be such that when it is multiplied by 3 and added to 241 you get a multiple of 11. This can be solved by a 'trial and improve' method. Using, in turn, 0, 1, 2, 3, ..., 9 for the unknown number, you can find the unique value which works.

$241 + 3 \times 0 = 241$ which is not a multiple of 11

$241 + 3 \times 1 = 244$ which is not a multiple of 11

$241 + 3 \times 2 = 247$ which is not a multiple of 11

$241 + 3 \times 3 = 250$ which is not a multiple of 11

$241 + 3 \times 4 = 253 = 23 \times 11$

So the unknown digit is correctly identified as 4.

Summary

This article has introduced the idea of error-detecting numerical codes. This involves adding extra numbers (a 'check digit'), mathematically structured to have a particular relation to the other numbers in the code, to the original number in order to allow mistakes of various sorts to be discovered. The two examples discussed were EAN barcodes for food and ISBNs for books, but these same ideas and features are very widely used in all sorts of codes.

Extract from A Suitable Boy by Vikram Seth

Powers of ten

Veena Tandon entered her house in Misri Mandi, to be greeted by her son Bhaskar with a kiss, which she happily accepted despite the fact that he had a cold. He then rushed back to the small sofa where he had been sitting—his father on one side and his father's guest on the other—and continued his explanation of the powers of ten.

Kedarnath Tandon looked at his son indulgently but, happy in the consciousness of Bhaskar's genius, did not pay much attention to what he was saying, His father's guest, Haresh Khanna, who had been introduced to Kedarnath by a mutual acquaintance in the shoe business, would have been happier talking about the leather and footwear trade of Brahmpur, but felt it best to indulge his host's son—especially as Bhaskar, carried away by his enthusiasm, would have been very disappointed to lose his indoor audience on a day when he had not been allowed to go out kite-flying. He tried to concentrate on what Bhaskar was saying.

'Well, you see, Haresh Chacha, it's like this. First you have ten, that's just ten, that is, ten to the first power. Then you have a hundred, which is ten times ten, which makes it ten to the second power. Then you have a thousand, which is ten to the third power. Then you have ten thousand, which is ten to the fourth power—but this is where the problem begins, don't you see? We don't have a special word for that—and we really should. Ten times that is ten to the fifth power, which is a lakh. Then we have ten to the sixth power, which is a million, ten to the seventh power which is a crore, and then we come to another power for which we don't have a word—which is ten to the eighth. We should have a word for that as well. Then ten to the ninth power is a billion, and then comes ten to the tenth. Now it's amazing that we don't have word in either English or Hindi for a number that is as important as ten to the tenth. Don't you agree with me, Haresh Chacha?' he continued, his bright eyes fixed on Haresh's face.

'But you know,' said Haresh, pulling something out of his recent memory for the enthusiastic Bhaskar, 'I think there is a special word for ten thousand. The Chinese tanners of Calcutta, with whom we have some dealings, once told me that they used the number ten-thousand as a standard unit of counting. What they call it I can't remember, but just as we use a lakh as a natural measuring point, they use ten-thousand.'

Bhaskar was electrified. 'But Haresh Chacha, you must find that number for me,' he said. 'You must find out what they call it. I have to know,' he said, his eyes burning with mystical fire, and his small frog-like features taking on an astonishing radiance.

'All right,' said Haresh. 'I'll tell you what. When I go back to Kanpur, I'll make enquiries, and as soon as I find out what that number is, I'll send you a letter. Who knows, perhaps they even have a number for ten to the eighth.'

'Do you really think so?' breathed Bhaskar wonderingly. His pleasure was akin to that of a stamp-collector who finds the two missing values in an incomplete series suddenly supplied to him by a total stranger. 'When are you going back to Kanpur?'

. . .

‘The trouble is that he has no one to talk to about his maths, and sometimes he becomes very withdrawn. His teachers at school are less proud of him than worried about him. Sometimes it seems he deliberately does badly in maths—if a question annoys him, for instance. Once, when he was very young, I remember Maan—that’s my brother—asked him for the answer to 17 minus 6. When he got 11, Maan asked him to subtract 6 again. When he got 5, Maan asked him to subtract 6 yet again. And Bhaskar actually began to cry! “No, no,” he said, “Maan Maama is playing a trick on me. Stop him!” And he wouldn’t speak to him for a week.’

‘Well, for a day or two at least,’ said Kedarnath. ‘But that was before he learned about negative numbers. Once he did, he insisted on taking bigger things away from smaller things the whole day long. I suppose, the way things are going with my work, he’ll get plenty of practice in that line.’

Source: Vikram Seth (1993) *A Suitable Boy*

Extract from Africa Counts: 'Games to grow on' by Claudia Zaslavsky

Counting rhymes and rhythms

Children in Africa, as in other parts of the world, learn finger counting rhymes even before they are aware of the number sequence. Some rhymes go only to five, while others continue as far as twenty. Most stop at ten, corresponding to the number of fingers. In some areas the rhymes are based on a twelve system or give special emphasis to multiples of three or four.

Here is a popular five-finger Swahili verse, with an element of daring, translated freely: 'Let's go!—Where?—To steal!—What about the police?—I'm out of this!' The counting begins with the little finger and ends at the thumb; with each phrase a finger is ticked off.

Many rhymes have nonsense words, or words that are obsolete or of foreign origin. The children of the Taita Hills, in southeastern Kenya, sing counting rhymes with words having no obvious meaning. According to John Williamson, an African adult made the discovery that some of these words had once been in the local language, but had been obsolete for a century. Amazingly, they were preserved in the children's songs. The children sing these words to a simple tune when they play games and even use the verses in their arithmetic work. In some districts they accompany the song by bending down the fingers of the right hand with those of the left, beginning with the little finger, and then continuing with the little finger of the left hand. These gestures are absolutely unrelated to the formal system of finger counting of the Taita people.

Counting songs are among the first items of the Venda children's musical repertory. They, too, accompany the words by counting on the fingers. Using their right index finger, they first tap the little finger of the left hand, then each consecutive finger until the thumb is reached. Counting on the right hand starts with the thumb and proceeds to the little finger, each in turn being grasped by the thumb and first finger of the left hand. Sometimes the child claps his hand when he reaches ten. The actual number words embrace several languages besides Venda—Thong, Sotho, Afrikaans, English.

Venda children use a counting song to choose a child to perform an unpleasant task—the last one is the loser. The children good humoredly shout 'Witch' at the odd child.

...

In a Shona game, played as they sit around the fire at night, the children must listen carefully and count accurately. While an older man recites certain verses in a rhythmic pattern, the children count the number of principal beats. The teacher may vary his speed, speaking rapidly, then slowly. Or he may enunciate the words very distinctly, so that the audience loses the rhythm. All this is accompanied by a great deal of laughter and clapping. The losers are subjected to some good-natured mocking at the hands of the star students.

Three in a row

Tic-tac-toe, three in a row. Several versions of this game are found in Africa, all more complicated than the familiar 'noughts and crosses'. It may be played on a board, or on lines drawn in the ground.

In Zimbabwe (Rhodesia) young men and boys play the game on a board (Figure 3a). Each of the two contestants has twelve stones; Player One's stones must be distinguishable from those of Player Two. There are three separate stages in the game.

In the first stage each player in turn places a stone at an intersection not already occupied by his opponent's stone. The object is to form a line of three counters in any direction. The player who succeeds in forming a line of three is entitled to remove any one of his opponent's stones. Once a player has formed a line of three, he may, on the next move, remove one of the stones in the line, and place it at a different intersection on his subsequent move, in order to capture another of the enemy's counters.

Stage two begins when each player has placed all twelve of his stones on the board. They then proceed to move the stones one space at a time along a line, again with the object of completing lines of three. Obviously some of the stones will have been captured; otherwise the twenty-four stones are locked into position at the twenty-four intersection points, and no move is possible.

The third stage occurs when one of the players has only three stones left. He may then move a stone to any free intersection point on the board. When one player is reduced to two stones, he has lost the game.

The version called 'African Morris' in the pamphlet *Ancient Games* (Cooperative Recreation Service) is played by the Asante, with rules similar to those in Zimbabwe. This game is indeed ancient. 3300 years ago playing 'board' was cut into a roof slab of the ancient Egyptian temple at Kurna. It has a cross in the centre square.

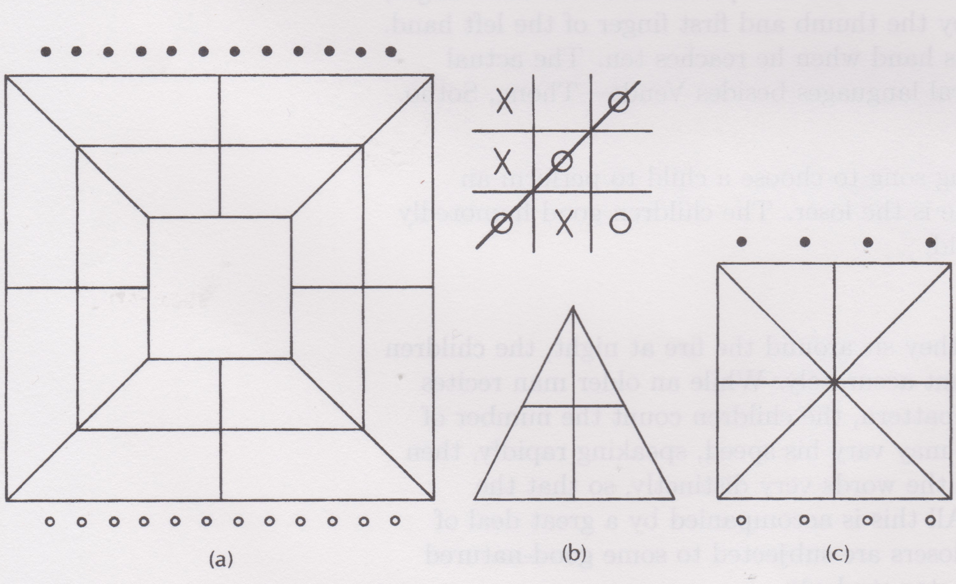

Figure 3 Several versions of the 'three-in-a-row' game are played in Africa, all more complicated than the familiar 'noughts and crosses.' Versions (a) and (b) are popular among boys and men in Zimbabwe (Rhodesia); (c) is played by Asante children.

A similar game, The Mill, or Nine Men's Morris, is described by Geoffrey Mott-Smith in *Mathematical Puzzles.* He calls it 'an ancient game that may have had a common origin with Tit-Tat-Toe (*sic*) ... As a subject of mathematical inquiry, The Mill is a fascinating combination of ideas. The basic three-in-a-row principle, completely exhausted in Tit-Tat-Toe, is given vitality in two ways. (a) The board is somewhat enlarged, but is still so confined that the player continually feels "With a little thought I could analyze this game completely!" (b) A game of placement is combined with game of movement. It is no longer sufficient to be the first to make "three in-a-row".'

Another Zimbabwe version is played on a triangular network: it is called *Tsoro Yematatu* (*tatu* means 'three'). There are seven intersections, and each player uses just three stones, which may be moved anywhere (Figure 3b). The first to complete a line of three is the winner. This is more difficult than it would seem at first glance, since there is only one free intersection available, once all the stones have been placed.

Asante schoolchildren play on a square network, marked on the ground (Figure 3c). Each player has four sticks or colored stones. After each player has placed his stones on the intersection points, one at a time he may move one of his markers one space along a line. The object, again is to place three stones in a row.

Networks

Early in this century the Belgian, Emil Torday, lived for some time among the Shongo people of the Congo area. One day he went up to a circle of small children playing with sand. 'I was invited to sit down, and one of them, Minge Bengela, divested himself of his loin cloth and offered it to me as a seat. This bettered Sir Walter Raleigh's action, as my young gallant was devoid of all other clothing. The children were drawing, and I was at once asked to perform certain impossible tasks; great was their joy when the white man failed to accomplish them.'

The task was to draw each of the figures in the sand without lifting the finger or retracing any line segment (Figure 4).

You may have tried to draw more simple figures or networks without lifting your pen or retracing. Try those in Figure 5.

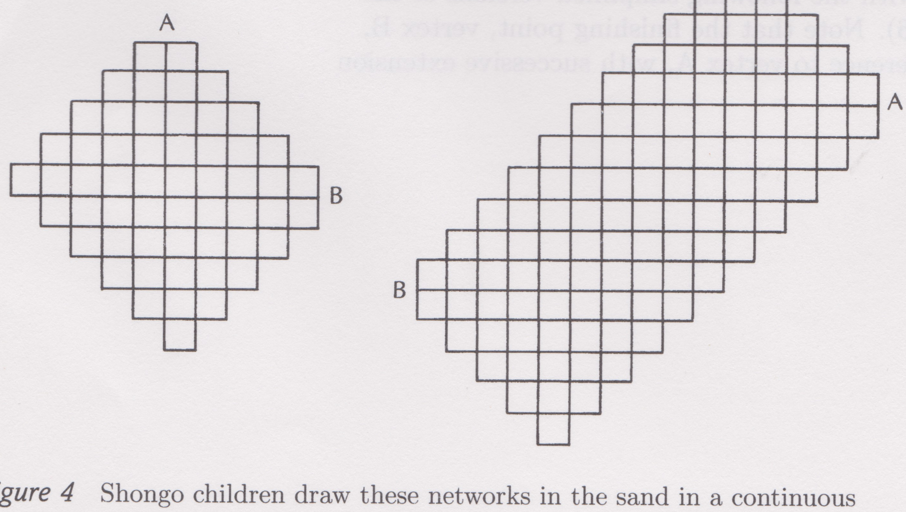

Figure 4 Shongo children draw these networks in the sand in a continuous line, without lifting the finger (after Torday).

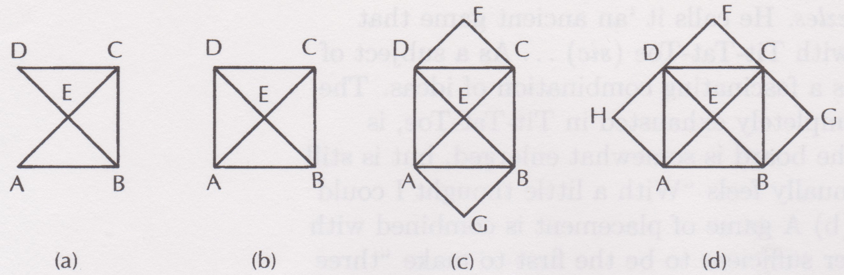

Figure 5

Networks a, c and d can be done, but network b is impossible. To determine whether a figure is traceable, we must examine the vertices to see how many are odd and how many are even. A vertex is a point at which two or more line segments meet. Let us analyze network a, which has five vertices. Two line segments meet at vertices A and D, three at B and C, and four at E. Vertices B and C are called odd vertices, since each is the meeting point of an odd number of line segments, and A, D, and E are called even vertices.

We can make a table of the number of line segments which meet at each vertex (Table 1):

Table 1 Number of line segments meeting at each vertex

Figure	A	B	C	D	E	F	G	H	Number of odd vertices	Traceable?
a	2	3	3	2	4	–	–	–	two	yes
b	3	3	3	3	4	–	–	–	four	no
c	4	4	4	4	4	2	2	–	none	yes
d	4	4	5	5	4	2	2	2	two	yes

Any network with exactly two odd vertices is traversable by a single path—can be drawn without lifting the pen or retracing. One of the odd vertices is the starting point of the path, and the other is the stopping point. A network which has no odd vertices at all, such as figure c, can be traversed by a single path no matter where one starts.

Now we can return to the Shongo children with the knowledge that the task is not impossible, since each figure has two odd vertices, at A and at B. You may want to start with the following simplified versions of the two Shongo networks (Figure 6). Note that the finishing point, vertex B, alternates its position with reference to vertex A, with successive extension of the network.

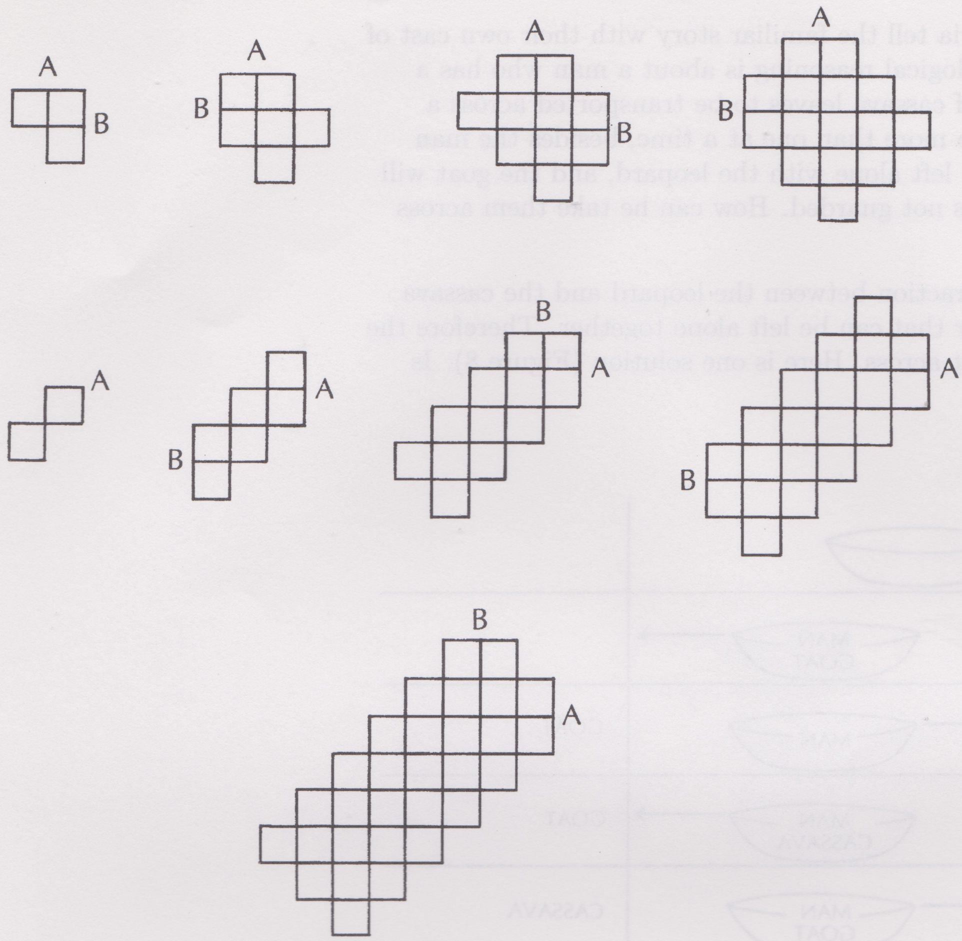

Figure 6 Simplified versions of the Shongo networks

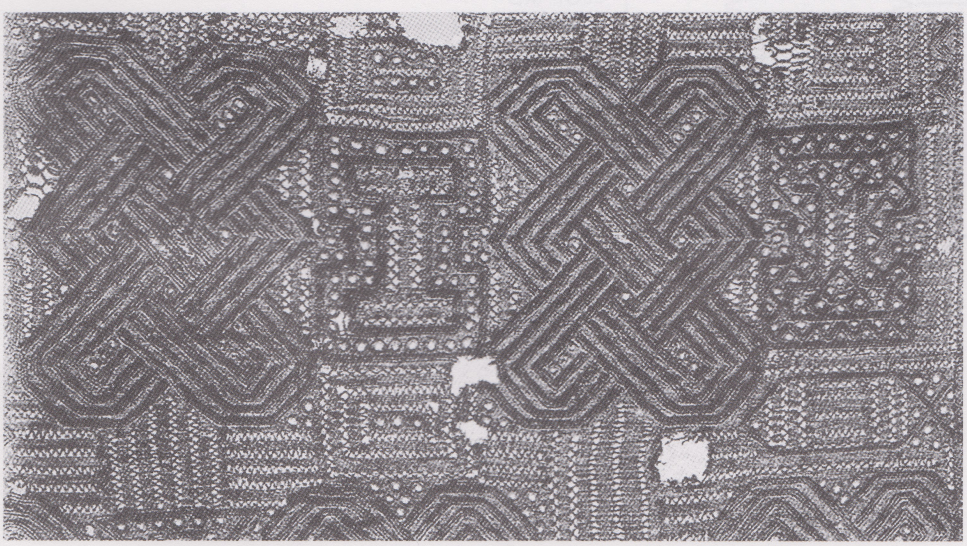

Figure 7 Kuba (Shongo) embroidered raffia cloth, Zaire (Congo). The interlacing *mbolo* pattern is similar to those drawn in the sand by Kuba children. This cloth dates back to the eighteenth century (British Museum).

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Riddles

The Kpelle children of Liberia tell the familiar story with their own cast of characters. This exercise in logical reasoning is about a man who has a leopard, a goat, and a pile of cassava leaves to be transported across a river. The boat can carry no more than one at a time, besides the man himself. The goat cannot be left alone with the leopard, and the goat will eat the cassava leaves if he is not guarded. How can he take them across the river?

Since there is no mutual attraction between the leopard and the cassava leaves, they are the only pair that can be left alone together. Therefore the man must first ferry the goat across. Here is one solution (Figure 8). Is there another?

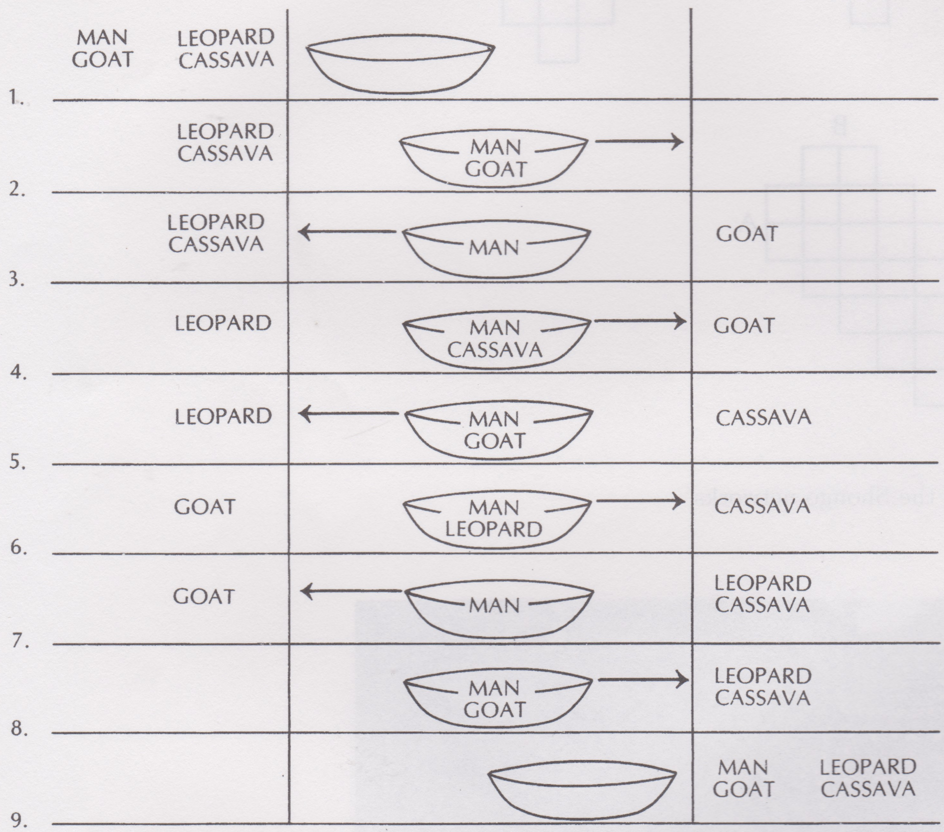

Figure 8

Arrangements

Many children's games depend upon an arrangement of objects. The Kpelle line up sixteen stones in two rows of eight each. One person is sent away, and the others choose a stone, When the 'out' person returns, he must determine which stone has been selected. He may ask four times in which of the two rows the stone is located. After each reply, he may rearrange the stones within the two rows. He must be able to identify the chosen stone after the fourth reply.

The key to the solution lies in the procedure by which the stones are rearranged each time. After the first reply the questioner rearranges eight stones, half the original number, so that they are now in different rows; after the second reply he interchanges four stones, half the previous number, and the next time he changes the positions of two stones. The answer to the last question determines precisely which stone has been chosen.

Initial Position									
	Row 1	1	2	3	4	5	6	7	8
	Row 2	9	10	11	12	13	14	15	16

Suppose 6 is the chosen stone. Here is one procedure the questioner may use:

Answer to question 1: 'It is in the first row.'
He interchanges the odd-numbered stones:
$1 \longleftrightarrow 9$, $3 \longleftrightarrow 11$, $5 \longleftrightarrow 13$, $7 \longleftrightarrow 15$

Second Position									
	Row 1	9	2	11	4	13	6	15	8
	Row 2	1	10	3	12	5	14	7	16

Answer to question 2: 'It is in the first row.'
He knows that the stone is even-numbered. He interchanges stones numbered $4n$ (multiples of four): $4 \longleftrightarrow 12$, $8 \longleftrightarrow 16$

Third Position									
	Row 1	9	2	11	12	13	6	15	16
	Row 2	1	10	3	4	5	14	7	8

Answer to question 3: 'It is in the first row.'
The chosen stone must be 2 or 6. He interchanges 6 and 14.

Fourth Position									
	Row 1	9	2	11	12	13	14	15	16
	Row 2	1	10	3	4	5	6	7	8

Answer to question 4: 'It is now in the second row'. The only candidate is stone 6. That, of course, is the correct answer.

Children in northeastern Tanzania play *tarumbeta*, so called because the pattern by the beans resembles a trumpet. Forty-five beans are arranged in nine rows to form a triangle (Figure 9). Four boys participate. The 'chief' sits at the apex, and serves as umpire, while the challenger sits at the base with his back to the triangle. The other two boys sit on each side of the triangle and remove one bean at a time, in turn, starting at the base and working toward the apex. After each move the challenger must give the number name of the bean just removed. However, whenever the first bean of any row has been picked up, he is forbidden to answer. Of course, he is not permitted to look at the triangle, but must visualize the position of the beans after each move. Youngsters are trained for such games by their older playmates. Small children may start with arrangements of ten

objects, and work up to the required number. In the case of ten objects, the challenger calls out: '(silence), 4, 2, 3, (silence), 7, 6, (silence), 8, 10.' Now try it with the triangle of forty-five objects arranged in nine rows!

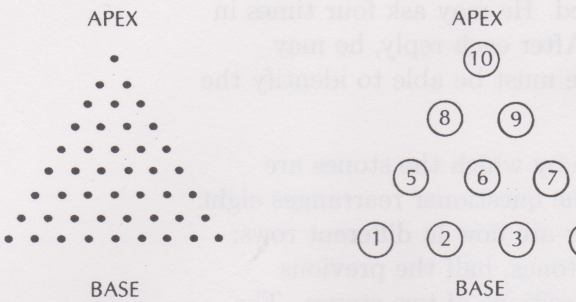

Figure 9 Children in northeastern Tanzania play *tarumbeta*, involving a triangular array of forty-five stones. Young children are trained with arrays of ten stones.

. . .

[Please note that 'Shongo' people are now called 'Bushoong'.]

Source: Claudia Zaslavsky (1973) *Africa Counts*

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